Tall-Structure Lightning Return-Stroke Modelling

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Abstract—A lightning return-stroke model, based on the lightning return-stroke current derivative measured at the Toronto CN Tower is presented in this paper. A 3-section transmission line representation of the CN Tower along with the derivative of the modified Heidler function is used to simulate the measured current derivative signal. Reflections from major structural discontinuities of the CN Tower as well as reflections from the upward-propagating lightning return-stroke channel front are taken into account in the proposed model. The proposed return-stroke model is experimentally evaluated based on the comparison between the simulated and measured magnetic field.

Index Terms – Tall-structure lightning, lightning current, return-stroke models, lightning electromagnetic pulse

I. INTRODUCTION

One of the most important parameters of the lightning discharge that is of interest to researchers, especially from the point of view of protection, is the lightning return-stroke current, because it usually causes most of the damage and interruption to power lines. Obviously, in order to measure the lightning current, the exact location of the lightning strike must be known, which is possible at tall-instrumented towers and towers placed on elevated grounds, or at rocket-triggered lightning stations. Since the lightning current is rarely measured, the lightning current characteristics are estimated based on the measured lightning-generated electromagnetic pulse (LEMP) through the use of a lightning return-stroke model.

Simultaneous measurements of the lightning current and its generated electromagnetic field, as well as the recording of the characteristics of its associated lightning channel, are necessary for the quantitative assessment of lightning return-stroke models. Such assessment will enhance our understanding of the lightning return-stroke process, help in the formulation of new models that can be more successfully used in lightning detection algorithms, and permit the development of sophisticated measures for lightning protection [1]. Also, the lightning-generated electromagnetic field, determined through the use of a specific return-stroke model, constitutes the basis for the calculation of lightning-induced overvoltages on power lines [2].

A classical lightning return-stroke model is a mathematical formulation that specifies the spatial-temporal distribution of the current within the lightning channel terminated on flat and ideal ground, thus allowing for the computation of LEMP.

For lightning terminating on an elevated object, the current distribution along the elevated object must also be determined. Simultaneous measurements of the lightning return-stroke current derivative at the 553-m CN Tower and the corresponding electromagnetic field at a distance of 2 km north of the Tower enabled the direct comparison between the measured LEMP and the computed one using a particular lightning return-stroke model [3].

The model developed in this paper is an engineering model [4] that is based on the lightning return-stroke current derivative measured at the CN Tower. The Tower is represented as a 3-section uniform transmission line – each section has different characteristic impedance – and the derivative of the modified Heidler function is used to simulate the measured current derivative signal. The simulated current derivative signal is used to determine the spatial-temporal distribution of the lightning current along the CN Tower and within the lightning channel, during the lightning return-stroke phase. Current reflections at the four major structural discontinuities of the CN Tower (tip, top and bottom of the restaurant, and ground) are thoroughly studied and their coefficients are determined and used in modelling. Also, the current reflection from the front of the upward-propagating lightning return-stroke channel front is considered and its coefficient is determined by investigating the measured current derivative signal. (In previous studies, the reflection coefficient at the front of the upward-propagating channel was roughly estimated.) Once the current distribution along the Tower and the lightning channel are determined, the magnetic field at any distance from the Tower can be computed. The comparison between the computed magnetic field, obtained through the use of Maxwell’s equations and the simulated current, and the measured magnetic field is carried out for the quantitative assessment of the proposed model.

The work presented in this paper has important applications in lightning protection, especially at tall structure, as well as in lightning detection algorithms, including the estimation of the lightning current based on the characteristics of the measured LEMP.
II. CN TOWER LIGHTNING MEASUREMENT SYSTEMS

Lightning strikes to the CN Tower have been observed for 32 years. Since 1991, five recording stations have operated to simultaneously capture important CN Tower lightning parameters, namely, the return-stroke current derivative at the 474-m above ground level (AGL) at the Tower (using a Rogowski coil), the vertical component of the electric field and the azimuthal component of the magnetic field 2 km north of the Tower (using broadband active sensors), the return-stroke velocity (RSV), and two 2-dimensional images (taken from directions that are approximately perpendicular to each other) of the flash trajectory (using VHS cameras), Fig. 1.

Since 1996, an expansion of the measurement facilities has been taking place. A 1000-frame/sec high-speed camera (HSC) was acquired in 1996 (Fig. 1). In 1997, a noise-protected current sensing system consisting of a new Rogowski coil and an optical fiber link was installed at the 509-m AGL at the Tower (Fig. 1). In 2001, two double-channel digitizers with 2-ns time resolution and large segmented memory were acquired to record the lightning return-stroke current derivative at the Tower and its corresponding electromagnetic field. For time synchronization of CN Tower lightning recording instruments, four global positioning system (GPS) units were also acquired. A more detailed description of the CN Tower lightning measurement system is included in [1].

The work presented in this paper is based on the measured lightning current derivative signal shown in Fig. 2. The figure also shows the corresponding current obtained by numerical integration. It is worth mentioning that the signal shown in Fig. 2 was captured by the old Rogowski coil.

The vertical component of the electric field ($E_z$) and the azimuthal component of the magnetic field ($H_\phi$), resulting from lightning strikes to the CN Tower and strikes in its vicinity, have been measured by broadband active field sensors [1]. The electric field sensor is an active, hollow hemispherical monopole with sensitivity of 1.44 V/(kV/m) and the magnetic field sensor is of the small-loop antenna type with sensitivity of 0.166 V/(A/m). The sensors are located 2 km north of the Tower, and are connected to an 8-bit, 4-ns LeCroy digitizer via coaxial cables.

Fig. 3 shows the vertical component of the electric field and the azimuthal component of the magnetic field of the LEMP measured 2 km north of the CN Tower, which were matched with the CN Tower lightning return-stroke current derivative and current waveforms shown in Fig. 2.
III. THE DERIVATIVE OF HEIDLER FUNCTION

Heidler function is a mathematical function that was designed to simulate the lightning return-stroke current [5]. Heidler function and its modified form are given by (1) and (2), respectively. The derivative of modified Heidler function is shown in (3). Parameter $A_2$ was eliminated by taking the second derivative of (3) and equating it to zero at $t = t_a$, where $t_a$ is the time of the peak of the initial impulse of the measured current derivative [3]. The modified Heidler function parameters, included in (2) and (3), are: $A_1$ and $A_2$ (control the channel base current impulse amplitude), $\tau_1$ and $\tau_3$ (front time constants), $\tau_2$ and $\tau_4$ (decay time constants), and $k_1$ and $k_2$ (exponents with values in the range $1.1 – 20$). The minimum value of $k_1$ or $k_2$ was chosen to be 1.1 in order to eliminate the discontinuity of the current derivative, given by (3), at $t = 0$.

$$i(t) = A_1 \left( \frac{(t/\tau_1)^{k_1} e^{-t/\tau_2}}{1 + (t/\tau_1)^{k_2}} \right)$$

$$i_{\text{mod}}(t) = A_1 \left[ \frac{(t/\tau_1)^{k_1} e^{-t/\tau_2}}{1 + (t/\tau_1)^{k_2}} \right] + A_2 \left[ \frac{(t/\tau_3)^{k_2} e^{-t/\tau_4}}{1 + (t/\tau_3)^{k_1}} \right]$$

$$\frac{di_{\text{mod}}(t)}{dt} = A_1 \left( \frac{t/\tau_1}{\tau_1} \right)^{k_1} e^{-t/\tau_2} \left( \frac{k_1 \tau_2 - t - t(t/\tau_1)^{k_1}}{t \left[ 1 + (t/\tau_1)^{k_1} \right]^2} \right)$$

$$+ A_2 \left( \frac{t/\tau_3}{\tau_3} \right)^{k_2} e^{-t/\tau_4} \left( \frac{k_2 \tau_4 - t - t(t/\tau_3)^{k_2}}{t \left[ 1 + (t/\tau_3)^{k_2} \right]^2} \right)$$

IV. INITIAL CURRENT DERIVATIVE IMPULSE

The first step in simulating the lightning current derivative involves nonlinear curve fitting of the initial impulse of the measured current derivative (before the arrival of any reflection) with the derivative of the modified Heidler function. The curve fitting was performed using Curve Fitting Toolbox from Matlab software package. The result of the curve fitting is shown in Fig. 4. The value of the R-Square, indicating the quality of fit or how well the regression line approximates the experimental points, obtained from curve fitting of the initial impulse was found to be 0.994. Figure 4 shows that the modified Heidler function (dashed line) matches very well the initial impulse of the measured current derivative with the exception of the initial stage ($0 - 0.18 \mu s$). The inability of the simulation function to accurately represent the initial stage of the measured current derivative deserves some attention [3].

V. REFLECTION COEFFICIENTS

The four main structural discontinuities of the CN Tower are the tip of the Tower, top and bottom of the restaurant and ground (Fig. 5). These structural discontinuities along the current path cause various reflections to appear in the measured current derivative signal (Fig. 2). In order to take into account these discontinuities, the Tower is modeled as a 3-section uniform transmission line (TL) to simulate the lightning current derivative in the range from 0 to 7 $\mu s$. The first section of the TL model exists between the Tower’s tip and the top of the restaurant; the second section is between the top and bottom of the restaurant, while the third section is between the bottom of the restaurant and ground. Furthermore, a 4th section of the transmission line is between the tip of the Tower and the continuously moving lightning return-stroke channel front.
In order to correctly determine the exact locations of the Tower’s structural discontinuities and the values of the reflection coefficients at those locations, an analysis was carried out trying to find correlations between these parameters and the rate of rise (steepness) of the measured signals. We used about 50 lightning return-stroke current derivative signals, measured at the Tower, to determine the locations where current reflections occur based on the assumption that the current waves propagate along the Tower at the speed of light. This analysis enabled the accurate determination of locations of reflections and the current reflection coefficients, which are essential for the determination of the current distribution along the lightning current path. It also simplified the calculation of the current reflection coefficients for the signal shown in Fig. 2.

Furthermore, the reflection coefficient at the top of the upward-propagating lightning return-stroke channel front was obtained by analyzing the current derivative waveform shown in Fig. 2. It was determined that a negative pulse seen in the measured current derivative waveform at about 5.85 μs (Fig. 2) corresponds to the ground reflection that is reflected back from the channel front. Knowing the reflection coefficient at the tip of the CN Tower, the reflection coefficient at the channel front was found to be $-0.268$.

\section*{VI. RETURN-STROKE VELOCITY}

In order to include the effect of channel front reflections on the simulated current derivative and current waveforms, the return-stroke speed in the channel has to be known. The initial speed of the return-stroke in the lightning channel was determined to be $0.32c$, based on the analysis of the measured current derivative signal (Fig. 2). Measurements of the CN Tower lighting return-stroke velocity (the propagation velocity of the channel front) were conducted in the past using a photodiode system. The system covered the vertical range of up to 180 m above the tip of the CN Tower and 20 m below the tip of the CN Tower. Return-stroke speeds in the range of $0.29c$ to $0.48c$ were found based on 45 flashes that struck the CN Tower during 1991 and 1992 lightning seasons [7]. It has also been noticed that the speed of the upward-propagating return-stroke channel front decreases as the channel front propagates in the upward direction above the tip of the CN Tower. In a study of rocket-triggered lightning channels, the speed of propagation of the return-stroke was also found to decrease as the height of channel front increases [8].

\section*{VII. CURRENT SIMULATION}

In this section, comparisons made between the simulated current derivative and current waveforms, and the measured current derivative and current waveforms, respectively. The derivative of the modified Heidler function was used for simulating the initial impulse of the measured current derivative (Fig. 4). Knowing the locations (tip, top and bottom of the restaurant and ground) and coefficients of current reflections within the Tower and at the upward-propagating return-stroke channel front, the simulated current derivative and current waveforms were computed by taking all reflections into account. The results presented in Figs. 6 and 7 show that both the simulated current and current derivative waveforms are well matched to the corresponding waveforms obtained from the measurement. Although the simulated current waveform, seen in Fig. 7, has a faster decaying portion in comparison to the measured current waveform, the main characteristics of the measured current are well reproduced by the simulation. For example, the simulated maximum current derivative only exceeds that of the measured signal by 1.5%. Also, the initial peak of the simulated current is only 2.4% larger than the measured initial current peak.

In future work, additional analysis of the decaying portion of the measured current has to be performed in order to improve the simulation results. Also, the possible modification of Heidler function or its modified form can be considered in order to improve the matching. Furthermore, it is recommended that minor structural discontinuities at the Tower are included in the TL model, which is expected to improve the simulation.

\section*{VIII. MAGNETIC FIELD SIMULATION}

As with every lightning return-stroke model, the proper validation of a proposed model is required. Proper model validation is accomplished by comparing the computed field, determined using the simulated current
and Maxwell’s equations [3], with the corresponding measured field. The derivative of Heidler function, along with a 3-section TL model of the CN Tower, is used to compute the magnetic field at a distance of 2 km from the Tower. The computed azimuthal component of the magnetic field ($H_\phi$) is compared with the measured one in Fig. 8. The computed magnetic field includes the radiation as well as the induction components. The reflections within CN Tower as well as the reflections from the upward-propagating lightning return-stroke channel front are included in the computation of the magnetic field. In order to view the effect of including reflections at the continuously propagating channel-front, a comparison between the computed magnetic field with and without channel reflections is shown in Fig. 9. The figure shows a marked effect at around 6 μs, resulting from the ground reflection when it reflects back from the channel front. Also, a lower magnetic field is noticed at 2.54 μs, due to the top of restaurant reflection when it reflects back from the channel front. The expected decrease in the return-stroke velocity is determined from the measured current derivative signal (Fig. 2). The initial speed until the top of restaurant reflection reaches the channel front was found to be 0.32c, whereas it was estimated to be 0.2c after that.

The initial peak of the simulated magnetic field (Fig. 8) is 1 [A/m] while the initial peak of the measured magnetic field is 1.42 [A/m]. Therefore, the initial simulated peak of the magnetic field is 70% of the measured peak. This difference between the simulated and measured field peaks could be attributed to a higher return-stroke velocity than the one obtained from the measured current derivative signal, especially near the Tower’s tip. Initial return stroke velocity very much affects the initial peak of the generated LEMP. Also, using a thin-wire antenna model to simulate the lightning return-stroke current using a different CN Tower lightning stroke resulted in a lower simulated initial magnetic field peak in comparison with the measured peak [9].

Furthermore, the clearly visible split peak in the measured field is not produced in the computed field. This split peak is likely related to a minor structural discontinuity near the Tower’s tip, which was not taken into consideration in this work. Also the near zero crossing of the measured magnetic field around 4 μs was not produced, which may relate to building enhancement effects or the assumption of having a perfect conducting ground along the propagation path.
IX. CONCLUSIONS

The most important parameter of the lightning return-stroke is the current, especially from the point of view of protection. The measurement of the lightning current presents a challenge since in order to measure the current the exact location of the lightning strike has to be known. In this paper, we presented a lightning return-stroke model based on the derivative of the modified Heidler function and a multi-reflection transmission line representation of the CN Tower and the attached lightning return-stroke channel. Current reflections from four main structural discontinuities within the Tower as well as from the upward-propagating channel front were taken into consideration. Locations of discontinuities as well as current reflection coefficients were obtained from the measured current derivative signal. The reflections from the channel front were included in the simulation of the current derivative and the current, as well as the associated magnetic field. The reflection coefficient at the top of the upward-propagating lightning return-stroke channel front was determined based on the detailed analysis of the measured current derivative.

The derivative of the modified Heidler function was first curve fitted to the initial impulse of the measured current derivative signal, before the arrival of reflections. Applying a multi-reflection transmission line procedure, the simulation of the whole measured current derivative signal was accomplished as a first step towards the computation of the generated magnetic field.

The peak of the simulated magnetic field was found to be 70% of the measured one. The discrepancy between the simulated and measured field peaks, which was noticed in earlier work using a thin-wire antenna model, is mainly attributed to the anticipated higher return stroke velocity during the formation of the field peak. Also, the split peak in the measured field, which was not produced in the computed field, is likely related to a minor structural discontinuity near the Tower’s tip, which was not taken into consideration.

REFERENCES


Mariusz Milewski was born in Warsaw, Poland. He received his B. Eng. degree in electrical engineering from Ryerson University, Toronto, Canada in 1999 and his M.E.Sc. degree in electrical engineering from the University of Western Ontario, London, Canada in 2003. In 2010 he received his Ph.D. degree in electrical engineering from Ryerson University, Toronto, Canada. From 2002 to 2009, he was a research assistant in the Department of Electrical and Computer Engineering, Ryerson University. Since January 2010, he has been a Post Doctoral Fellow in the Department of Electrical and Computer Engineering, Ryerson University. He has co-authored about 30 publications in the area of tall-structure lightning and lightning electromagnetic pulse (LEMP).

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