Development of New Fuzzy Logic-based Ant Colony Optimization Algorithm for Combinatorial Problems

Ahmed Rabie Ginidi Ginidi
Automatic Control and System Engineering Group
Dept. of Electric Power and Machines Engineering
Faculty of Engineering, Cairo University, Giza, Egypt
ahmedrabi@gyendi@yahoo.com

Ahmed M. A. M. Kamel
Automatic Control and System Engineering Group
Dept. of Electric Power and Machines Engineering
Faculty of Engineering, Cairo University, Giza, Egypt
amakamel@hotmail.com

Hassen Taher Dorrah
Automatic Control and System Engineering Group
Dept. of Electric Power and Machines Engineering
Faculty of Engineering, Cairo University, Giza, Egypt
dorrahht@aol.com

Abstract - This paper is directed towards developing a new fuzzy-logic based ACO algorithm. The proposed algorithm takes into consideration the uncertainties that can be found in both the heuristic and the pheromone factors. This is achieved by representing the parameters of the problem and the pheromone trails as a pair of value and fuzzy level. The fuzzy level is considered as an indication of the uncertainty in the corresponding parameter. During the solution iterations the calculations are performed taking into consideration the fuzzy levels of the involved parameters. Hence, both pheromone updates and heuristic information are calculated with their corresponding fuzzy levels. Consequently, the probabilities of choosing the possible oncoming step are also calculated with their corresponding fuzzy levels. A stochastic-based technique is proposed to enable the artificial ant to choose the best oncoming step based on the values of the probabilities and their corresponding fuzzy levels.

The proposed Fuzzy Ant Colony Optimization (FACO) gives the optimal solution in a form of an optimal value and its corresponding fuzzy level. The fuzzy level of the solution is interpreted as the uncertainty in the value of the optimal result due to the uncertainties of the pheromone trails and the problem parameters. The proposed FACO algorithm is tested using the benchmark Quadratic Assignment Problem (QAP) and Travelling Salesman Problem (TSP). The results indicate that the developed FACO has better values with improved performance.

Index Terms – Ant Colony Algorithm, Quadratic Assignment Problem, Travelling Salesman Problem, Combinatorial Problems, Fuzzy Systems, Fuzzy-based Logic Algebra

I. INTRODUCTION

Combinatorial optimization problems involve finding values for discrete variables such that the optimal solution with respect to a given objective function is found. Many optimization problems of practical and theoretical importance are of combinatorial nature. Examples are the shortest-path problems, as well as many other important real-world problems like finding minimum cost plan to deliver goods to customers, an optimal assignment of employees to tasks to be performed, a best routing scheme for data packet in the Internet, an optimal sequence of jobs which are to be processed in a production line, an allocation of flight crews to airplanes, and many more.

Combinatorial problems are intriguing because they are often easy to state but very difficult to solve. Some of the best known and widely applied metaheuristics are simulated annealing, tabu search, evolutionary computation,…etc have been developed to solve these problems.

One of the important classes of combinatorial optimization problems is the quadratic assignment problem (QAP). In this problem facilities are to be assigned to locations to optimize certain cost function. Some fields of applications of this problem are planning, analyzing, solving networks and control of the realization of complex projects. Several methods such as particle swarm [1], genetic algorithm [2], and tabu search [3] have been developed to solve these problems. Another class of the combinatorial optimization problems is the class of the travelling salesman problems (TSP). In these problems, a sales man is required to make a complete tour through certain number of cities such that the tour achieves certain objective function. Several methods such as genetic algorithm [4] and ant colony optimization [5] have been developed to solve these problems.

Combinatorial optimization problems can be modeled by either deterministic or probabilistic (stochastic) representations. Conventional methods are based on exhaustive search, that is, the enumeration of all possible solutions and the choice of the best one. Unfortunately, in most cases, such a native approach becomes rapidly infeasible because the number of possible solutions grows exponentially with the problem.

The world of metaheuristics is rich and multifaceted. Several characteristics make ACO a unique approach: it is a constructive, population-based metaheuristic which exploits an indirect form of memory of previous performance. This combination of characteristics is not found in any of other metaheuristics. Consequently, ACO techniques are one of the recent and most successful heuristic methods [6, 7, 8].

Unfortunately, these algorithms have been developed for deterministic representation of the combinatorial problems. However, the practical systems, a need for developing efficient ACO technique that is able to represent all the uncertainties of the parameters of the system in a way suitable for performing calculations and giving optimal solution in reasonable time arises [9, 10, 11]. This paper introduces the development of such technique.
II. ANT COLONY OPTIMIZATION ALGORITHM

ACO is an evolutionary metaheuristic algorithm based on a graph representation that has been applied successfully to solve various hard combinatorial optimization problems. The main idea of ACO is to model the problem as the search for a minimum cost path in a graph. Artificial ants walk through this graph, looking for good paths. Each ant has a rather simple behavior so that it will typically only find rather poor-quality paths on its own. Better paths are found as the emergent result of the global cooperation among ants in the colony.

The behavior of artificial ants is inspired from real ants. They lay pheromone trails on the graph edges and choose their path with respect to probabilities that depend on pheromone trails and these pheromone trails progressively decrease by evaporation. Ants prefer to move to nodes, which are connected by short edges with a high amount of pheromone. In addition, artificial ants have some extra features that do not find their counterpart in real ants. In particular, they live in a discrete world and their moves consist of transitions from nodes to nodes as shown in fig. 1. Also, they are usually associated with data structures that contain the memory of their previous action. In most cases, the amount of pheromone deposited is usually a function of the quality of the path. Finally, the probability for an artificial ant to choose an edge often depends not only on pheromone, but also on some problem-specific local heuristics.

At each generation, each ant generates a complete tour by choosing the nodes according to a probabilistic state transition rule. Every ant selects the nodes in the order in which they will appear in the permutation. For the selection of a node, an ant uses a heuristic factor as well as a pheromone factor. The heuristic factor, denoted by \( \eta_{ij} \), and the pheromone factor, denoted by \( \tau_{ij} \), are indicators of how good it seems to have node \( j \) at node \( i \) of the permutation. The heuristic value is generated by some problem dependent heuristics whereas the pheromone factor stems from former ants that have found good solutions.

\( \eta_{ij} \) is the visibility function which is usually selected as the inverse of the weight for \( i \) to \( j \). Lower links are made more desirable, \( d_{ij} \) represents the weight between links, then ant prefers selection of the shortest links. i.e:

\[
\eta_{ij} = 1 / d_{ij}
\] (1)

To study the whole space of solutions, the pheromones should evaporate. If the coefficient of evaporation is denoted by \( \rho \in [0,1] \), the update rule for the pheromones takes the form

\[
\Delta \tau_{ij}(t) = \Delta \tau_{ij}(t) + Q / L_k(t)
\] (2)

The pheromone value on edges is taken to be equal to a small positive number \( r_0 \).

When the tour is completed, the \( K^{th} \) ant lays down on the edge \( (i, j) \) the pheromone value

\[
\Delta \tau_{ij}(t) = \begin{cases} (Q / L_k(t)), & \text{if } (i, j) \in T_k(t) \\ 0, & \text{if } (i, j) \notin T_k(t) \end{cases}
\] (3)

where \( L_k(t) \) is the solution of the ant \( m \) at iteration \( t \) (fitness value of solution), \( T_k(t) \) is the tour chosen by the ant and \( Q > 0 \) is an adjustable parameter. If the amount of pheromone deposited is inversely proportional to the quality of the solution, then the larger \( L_k(t) \) (that is, the worse the constructed solution), the smaller \( 1 / L_k(t) \), hence the less the amount of pheromone deposited on the link. Thus, a long path causes all the links of that path to become less desirable as a component of the final solution. This is the case for any quality measure that needs to be minimized.

To study the whole space of solutions, the pheromones should evaporate, causing ants to forget previous decisions. The total number of ants in the colony remains constant.

The operational mechanism of basic ant colony algorithm is based on the combination of positive feedback principle and a certain heuristic search technique. It can be brought to light from the transition probability formula, formulated as follows:

\[
P_{ij}^k(t) = \left[ \tau_{ij}^k(t) \right]^\alpha \left[ \eta_{ij} \right]^\beta / \sum_{l \in L} \left[ \tau_{ij}^k(t) \right]^\alpha \left[ \eta_{ij} \right]^\beta
\] (5)

where \( \alpha \geq 0 \) and \( \beta \geq 0 \) are adjustable parameters describing the weights of the pheromone trail and visibility when choosing the route. To choose these parameters, the following observations can be obtained.

1. High values of \( \alpha \) and low values of \( \beta \) will lead to bad solutions due to premature convergence because the global intelligence is over emphasized and the

![Fig. 1. Probability of real ant to choose the path.](image-url)
local heuristic is severely discounted. The ant behaviours are highly affected by pheromone experiences and reach convergent behavior quickly after only a few iterations.

2. Low values of $\alpha$ and high values of $\beta$ can obtain above-average quality solutions and the results with different runs are more consistent. Among these settings, the experimental results with $(\alpha = 1, \beta = 5)$ or $(\alpha = 2, \beta = 5)$ has the best performance.

3. When either $\alpha = 0$ or $\beta = 0$, the performance is significantly deteriorated. As for the case of $\alpha = 0$, no pheromone information is used, i.e. previous search experience is neglected. The search then degrades to a stochastic greedy search and ACO is reduced to a greedy heuristic algorithm which considers only the value of $\eta_{\alpha=0}$. The ants just ignore the global intelligence represented as pheromone trails and do not know about the quality of the previously constructed solutions, so there is no communication happening between the ants. As for the case of $\beta = 0$, ACO becomes an exploitation-prone search method which intensifies the search within a small neighbourhood of the best solution observed so far and has a very low probability of exploring new regions of the solution space and the attractiveness of moves is neglected. The heuristic information adds an explicit bias towards the most attractive solutions, and is therefore a problem-dependent function.

III. DEFINITION OF PROPOSED FUZZY LOGIC ARITHMETIC REPRESENTATION

As this new approach is based on assigning a certain fuzzy level to each parameter and coefficient, the study will present first the definition of the fuzzy logic arithmetic representation. Then, the algebra of these fuzzy representations is given for the scalar forms as represented by Gabr and Dorrah [11].

Let $X$ be a general scalar parameter comprising two main components; as follows:

$$X = X_0 + X_f$$  \hspace{1cm} (6)

where $X_0$ is the deterministic equivalence and $X_f$ is the fuzzy equivalence representing a small uncertainty or value tolerance in the parameter $X$.

$$X = X_0(1+\ell_x), \ell_x = X_f / X_0$$  \hspace{1cm} (7)

The proposed fuzzy logic arithmetic representation is expressed by replacing each parameter with a pair of parenthesizes, the first is the actual value and the second is corresponding fuzzy level, that is (Value, Fuzzy Level). This is similar to vector representation of parameters, where vectors are not added directly. A summary of the main fuzzy logic based algebraic operations are presented in Table I.

<table>
<thead>
<tr>
<th>Operation</th>
<th>Symbolic Representation of Operation</th>
<th>Resulting Values and Fuzzy Levels from Original Operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiplication</td>
<td>$XY\ldots Z = (X_0Y_0\ldots Z_0, \ell_x + \ell_y + \ldots, \ell_Z)$</td>
<td></td>
</tr>
<tr>
<td>Division</td>
<td>$XY / Z = (X_0Y_0 / Z_0, \ell_x + \ell_y - \ell_z)$</td>
<td></td>
</tr>
<tr>
<td>Addition and Subtraction</td>
<td>$X-Y+Z = X_0 - Y_0 + Z_0, \ell_w = (X_0\ell_x - Y_0\ell_y + Z_0\ell_z) / (X_0 - Y_0 + Z_0)$</td>
<td></td>
</tr>
<tr>
<td>Polynomials</td>
<td>$X^{n/m}, Y = (X^{n/m}_0, \ell_y)$ and $\ell_y = (n/m)\ell_x$</td>
<td></td>
</tr>
</tbody>
</table>

IV. THE DEVELOPED FUZZY LOGIC-BASED ACO ALGORITHM

Conventional ant colony algorithm uses the parameters of the problem, the heuristic information and the pheromone trails to solve the combinatorial optimization problems.

In most of the practical problems, the parameters cannot be considered as crisp because they usually have different types of uncertainties. In this paper, a new fuzzy logic-based ant colony optimization (FACO) is presented to solve the combinatorial problems with uncertainties to its parameters. These uncertainties should affect both the heuristic information and pheromone update factors of the algorithms. The uncertainties of the heuristic information and pheromone update factors are represented as fuzzy levels. This means that Heuristic information will be developed as follows.

The heuristic information is

$$\eta_y = 1 / (d_{\eta_y}, \ell_{d_y})$$  \hspace{1cm} (12)

Applying the rules of fuzzy logic arithmetic representation, then

$$(\eta_{\eta_y}, \ell_{\eta_y}) = (1 / d_{\eta_y}, -\ell_{d_y})$$  \hspace{1cm} (13)

where $\eta_{\eta_y}$ is the crisp value of heuristic information, $\ell_{\eta_y}$ is the fuzzy level for the value of heuristic information $d_{\eta_y}$ represents the weight between two nodes and $\ell_{d_y}$ represents the fuzzy level for the weight between two nodes.

When the tour is completed, the Kth ant lays down on the edge $(i, j)$ the pheromone value

$$(\Delta \tau_{\eta_y,k}(t), \ell_{\Delta \tau_{\eta_y,k}(t)}) = \begin{cases} \frac{Q}{L_{\eta_y}(t)} - \ell_{\Delta \tau_{\eta_y,k}(t)}, & \text{if } (i, j) \in T_k(t) \\ 0, & \text{if } (i, j) \not\in T_k(t) \end{cases}$$  \hspace{1cm} (14)

where $\Delta \tau_{\eta_y,k}(t)$ is the crisp value of quantity of pheromone deposited by an ant $k$ at iteration $t$, $\ell_{\Delta \tau_{\eta_y,k}(t)}$ is the fuzzy level for the quantity of pheromone of the ant $k$ at iteration $t$, $L_{\eta_y}(t)$ is the crisp value for function (solution) deposited by
an ant $k$ at iteration $t$, and $\ell_{tik}$ is the fuzzy level for the function (solution) of the ant $k$ at iteration $t$.

Thus, the crisp value of quantity of pheromone on the path is 

$$\Delta r_{ijk}(t) = \Delta r_{ijk}(t) + \frac{Q}{L_{ot}(t)}$$

Then, the fuzzy level for the quantity of pheromone on the path will be 

$$\ell_{x_{tik}}(t) = \frac{\Delta r_{ijk}(t)}{\Delta x_{tik}}(t) - Q\ell_{t_{ik}}(t) / L_{oi}(t)$$

Consequently, the crisp value for the pheromone update is 

$$r_{ijk}(t + 1) = (1 - \rho) r_{ijk}(t) + \Delta r_{ijk}(t)$$

At the early stage of the optimization process, the pheromone value on edges is taken to be equal to a small positive number $\tau^0$. The fuzzy level for the pheromone update equation is 

$$\ell_{x_{ik}}(t) = (1 - \rho) \ell_{x_{ik}}(t) + \Delta \ell_{x_{ik}}(t)$$

In the conventional ACO algorithm, each ant uses the pheromone trail to calculate the probability of transferring from its current position to each of its neighbor nodes.

In the proposed technique, each probability has its own fuzzy level. Now, the ant has to make a decision using fuzzy probabilities. Fuzzy probabilities mean that when the ant calculates the probabilities of choosing node $x_n$ as depicted in Fig. 2, this value has a corresponding fuzzy level according to (20).

Consequently, the crisp value for probability equation is 

$$P_{xy}^h(t) = [\ell_{xy}(t)]^a [\eta_{xy}]^b / \sum_{l \in J} [\ell_{xy}(t)]^a [\eta_{xy}]^b$$

Then the fuzzy level for the probability equation is 

$$\ell_{P_{xy}}(t) = \frac{[\alpha_{\ell_{xy}}] + [\beta_{\eta_{xy}}]}{\sum_{l \in J} ([\alpha_{\ell_{xy}}] + [\beta_{\eta_{xy}}]) [\ell_{xy}(t)]^a [\eta_{xy}]^b} / \sum_{l \in J} [\ell_{xy}(t)]^a [\eta_{xy}]^b$$

As every element treated as a crisp value and its corresponding fuzzy level, thus the ant can choose the next node according to (21) 

$$P_{xy}^h(t) = P_{xy}^h(t)(1 + \ell_{P_{xy}}(t)), \quad 0 \leq P_{xy}^h(t) \leq 1$$

This means that the probability $P_{x_{ik}}$ has its fuzzy level of $\ell_{P_{x_{ik}}}$ as depicted in Fig.1. The decision making algorithm under these fuzzy conditions is based on assuming that each of the calculated probabilities has a probability density function (PDF) of certain type spread between $P$ and $P(1 + \ell_p)$. Then, the decision of the ant will be based on the average of the PDF. The PDF may be Gaussian, Beta, linear, nonlinear,...etc. However, in this approach, the linear probability density function can be considered.

The probability density function of system fuzziness is assumed to be trapezoidal as depicted in Fig. 3. It is defined in the interval $[0, P_{\ell_p}]$. The relationship between $h_1, h_2, P_0$, and $\ell_p$ can be given by 

$$\frac{(h_1 + h_2)}{2} P_{\ell_p} = 1,$$

This is a direct result as the area under the PDF should equal to 1.

\[ y = \frac{h_1 - h_2}{P_{\ell_p}} x + h_2 \] (23)

Using the above equation, the average value can be calculated as follows.

$$\text{Average value} = \int_0^{P_{\ell_p}} \left( \frac{h_1 - h_2}{P_{\ell_p}} \right) x \ dx, \quad 0 \leq x \leq P_{\ell_p}$$

$$= \frac{2 P_{\ell_p} \ell_p}{6} + h_1 \frac{P_{\ell_p}^2}{6}$$ (25)

The special cases of (20) are given below:-

1. For $h_1 = h_2$

The PDF of the system fuzziness is uniform. Then, the average value is given by $P_0(1 + \lambda \ell_p)$, where $\lambda = 1/2$.
2. For \( h = 0 \), and from (17) \( h = 2 / P_0 \ell p \).

The PDF of the system fuzziness is triangle with negative slope and consequently the probability between two nodes that is used in the decision making = \( P_0 (1 + \lambda \ell p) \), where \( \lambda = 1/3 \).

3. For \( h = 0 \), and from (17) \( h = 2 / P_0 \ell p \).

The PDF of the system fuzziness is triangle with positive slope and consequently the probability between two nodes that is used in the decision making = \( P_0 (1 + \lambda \ell p) \), where \( \lambda = 2/3 \).

The above discussion reveals that the probability between two nodes is given by \( P_0 (1 + \lambda \ell p) \), where \( \lambda \in [1/3, 2/3] \).

V. THE APPLICATION OF THE DEVELOPED FUZZY BASED ANT COLONY OPTIMIZATION TO TRAVELLING SALESMAN PROBLEM

The TSP can be stated as the problem of finding a minimal time required to go through a tour which involves traffics, accidents...etc with constraints that visits at each town must be once. This problem with uncertainties to its parameters can be solved by applying the developed FACO algorithm.

The first step is to prepare the input data. The time taken to travel from town \( i \) to town \( j \) is be given by \( d_{ij} \). Each time is then assigned a fuzzy level. This fuzzy level depends on the nature of the traffics found on the path from town \( i \) to town \( j \) as well as other aspects such as the statistics of the accidents on the path.

The second step is to calculate the heuristic information. The result, of course, will have a crisp value and a corresponding fuzzy level.

The third step is to determine the initial values of the pheromone trail at each branch. These values will be in pairs. Each pair has a crisp value and a corresponding fuzzy level.

During the construction of the solution, the decision of an ant to travel from its current position to its next position depends on the proposed technique. That is to say, the probabilities of going from the current position to its neighbours are calculated with their fuzzy levels. Then, the averages are calculated using (26) with the given value of \( \lambda \). The next position of the ant is the position that has maximum average.

At the end of iteration, the pheromone trails and their corresponding fuzzy levels are updated. This is done in two steps. The first is the pheromone deposited as given in (15) and its corresponding fuzzy level according to (16). The second is the pheromone evaporation according to (17) and its corresponding fuzzy level according to (18). The iterations continue until stopping criterion is achieved.

At the end of the iteration, the tour time of each ant and its corresponding fuzzy level are calculated. Then, the minimum tour time is the best of this iteration. The optimal tour time is the minimum of the best tour time of the current iteration and the minimum tour time of all iterations.

The path which gives the tour time is given in (27) and its corresponding fuzzy level is given in (28).

The total time \( L_{ij} (t) = \sum_{i=1}^{n} d_{\pi(i)(i+1)} + d_{\pi(n)(i+1)} \) (27)

The fuzzy level for the total time \( (T_{ij}(t)) = (\sum_{i=1}^{n} d_{\pi(i)(i+1)} l d_{\pi(i+1)(i)} + d_{\pi(n)(i+1)} l d_{\pi(i+1)(i)}) / (\sum_{i=1}^{n} d_{\pi(i)(i+1)} + d_{\pi(n)(i+1)}) \) (28)

where \( d_{ij} \) is the crisp value for the time between town \( i \) and \( j \). \( \pi_{ij} \) gives the time of town \( i \) in the current solution \( \pi \in S(n) \) and \( S \) is the candidate solution. The term \( d_{\pi(i)(i+1)} \) describes the time contributions of simultaneously choosing the path between town \( i \) and \( j \) and \( (d_{\pi(i+1)(i)}) \) represents the fuzzy level of the choosing the path between town \( i \) and \( j \).

The parameters of the algorithm are chosen to be \( \alpha = 1, \beta = 5, m=30, \rho=0.1 \) and \( Q=100 \). In addition, the parameter \( \lambda \) should discussed in the previous section is changed from 0 to 1. The Oliver30 benchmark TSP which has 30 cities and described in [12] is used. The best time and the average time when \( \lambda = 0 \), at each iteration, are shown in Fig. 4. From the figure, the time of the shortest tour is 423.5129 min and its corresponding fuzzy level equals to 0.038. It is noticed that the corresponding fuzzy level depend on the calculations.

It is noted that, in the previous results, \( \lambda = 0 \) is taken to be zero. This means that the choice of the ant depends only on the calculated probability. Hence, in order to complete the results,
$\lambda$ should take values other than zero and the parameters of the algorithm are chosen to be $\alpha = 1, \beta = 5, m = 30, \rho = 0.1$ and $Q = 100$ as mentioned in case $\lambda = 0$. That is to say, the performance of the proposed technique should be tested when the choice of the ant depends on the calculated probability and its corresponding fuzzy level. As discussed in the above section, if the probability density function is triangular with negative slope, the value of the parameter $\lambda$ will be $1/3$. When this value of $\lambda$ is taken into consideration, the proposed algorithm gives the results shown in Fig. 5. As depicted in the Figure, the solution is improved to 420.0115 with corresponding fuzzy level equals to 0.6797. The Figure shows that the response of the solution also is improved. That is to say, the optimal solution is reached in a relatively small number of iterations. Also, the average time of the tour at each iteration has less fluctuations than the previous case which is shown in Fig. 3.

If the probability density function is triangle with positive slope, the value of $\lambda$ is $2/3$. When this value of $\lambda$ is taken into consideration, the proposed algorithm gives the results shown in Fig. 7. As depicted in the figure, the solution is improved to 418.488 with corresponding fuzzy level equals to 1.216. Also, the response of the solution is improved. Also, the fluctuations of the average time of the tour at each iteration are decreased.

Table II and Fig. 8 summarize the effect of the value of $\lambda$ on the solution of TSP with uncertainty to its parameters.

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>FACO</th>
<th>FACO</th>
<th>FACO</th>
<th>FACO</th>
<th>FACO</th>
<th>FACO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value (Min)</td>
<td>423.51</td>
<td>420.01</td>
<td>419.29</td>
<td>418.49</td>
<td>425.53</td>
<td>427.51</td>
</tr>
<tr>
<td>Fuzzy Level</td>
<td>0.038</td>
<td>0.68</td>
<td>0.51</td>
<td>1.22</td>
<td>0.64</td>
<td>1.44</td>
</tr>
</tbody>
</table>

Fig. 5. Best and average time at each iteration, where $\lambda = 0.333$.

Fig. 6. Best and average time at each iteration where $\lambda = 0.5$.

Fig. 7. Best and average time at each iteration where $\lambda = 0.667$.

Fig. 8. Effect of FACO on the solution of tsp with uncertainties to its parameters.
It is to be noted that the above results are better than those in [13], where genetic algorithms were applied to solve the Oliver30 problem; they could find a tour of length 424.635 min. The same result was often obtained by ant-cycle [4], which also found a tour of length 423.741 min.

VI. THE APPLICATION OF THE DEVELOPED FUZZY LOGIC-BASED ANT COLONY OPTIMIZATION TO QUADRATIC ASSIGNMENT PROBLEM

To apply the FACO metaheuristic to assignment problems, a first step is to map the problem on a construction graph \( G_c=(C,L) \), where \( C \) is the set of components (usually the components consists of all locations and all the facilities) and \( L \) is the set of connections that fully connects the graph. Transitions are from facilities to locations and vice versa. Typically, an ant first chooses facility, then a location to which to assign the facility, then another facility, and so forth, until all facilities have been assigned. Facilities and locations are chosen from the feasible neighbourhood, that is, from facilities (locations) not signed yet. These constraints can be easily enforced in the ants’ walk by building only coupling between still unsigned facilities and locations.

The function can be described in (29) and its corresponding fuzzy level is given in (30).

\[
f(\pi) = \sum_{i}^{n} \sum_{j}^{n} b_{ij} a_{b_{ij}} \tag{29}
\]

\[
\ell_{f(\pi)} = \sum_{i=1}^{n} \sum_{j=1}^{n} b_{ij} a_{b_{ij}} (\ell_{f} + \ell_{a_{b_{ij}}}) / \sum_{i=1}^{n} \sum_{j=1}^{n} b_{ij} a_{b_{ij}} \tag{30}
\]

where \( b_{ij} \) is the crisp value for the flow between facilities \( i \) and \( j \), \( a_{b_{ij}} \) is the crisp value for the time between locations \( i \) and \( j \), \( \pi_{i} \) gives the location of facility \( i \) in the current solution \( \pi \in S(n) \) and \( S \) is the candidate solution. The term \( b_{ij} a_{b_{ij}} \) describes the cost contributions of simultaneously assigning facility \( i \) to location \( \pi_{i} \) and facility \( j \) to location \( \pi_{j} \) and \((\ell_{f} + \ell_{a_{b_{ij}}})\) represents the fuzzy level of the cost contributions of simultaneously assigning facility \( i \) to location \( \pi_{i} \) and facility \( j \) to location \( \pi_{j} \).

This problem with uncertainties to its parameters can be solved by applying the developed FACO algorithm.

The first step is to prepare the input data. The input parameters are the time between locations, the flow between facilities and the parameters of Ant Colony Technique. Each time and each flow is then assigned a fuzzy level.

The second step is to calculate the heuristic information. The result, of course, will have a crisp value and a corresponding fuzzy level. Calculating this heuristic information on the potential goodness of an assignment is as follows. Two vectors \( d \) and \( f \) are calculated in which the \( i \)th components represent respectively the sum of distances from location \( i \) to all other locations, and the sum of the flows from facility \( i \) to all other facilities.

For example, the lower \( d_{i} \), the time potential of location \( i \), the more central the location, the higher \( f_{j} \), the flow potential of facility \( i \), the more important is the facility. Next a coupling matrix \( E = f_{i}d_{j}^{T} \) is calculated, whose elements are \( e_{ij} = f_{i}d_{j} \). Then, the heuristic desirability of assigning facility \( i \) to \( j \) is given by \( \eta_{ij} = 1/e_{ij} \). The motivation for using this type of heuristic information is that, intuitively, good solutions will place facilities with high flow potential on locations with low time potential.

\[
d_{i} = \sum_{j=1}^{n} D_{ij} \forall j = 1 \rightarrow n \quad i \neq j \tag{31}
\]

The corresponding fuzzy level for the sum of times from location \( d \) to all other

\[
(\ell_{d}) = \sum_{j=1}^{n} D_{ij} \ell_{d_{j}} / \sum_{j=1}^{n} D_{ij} \forall j = 1 \rightarrow n \quad i \neq j \tag{32}
\]

The corresponding fuzzy level for the elements of coupling matrix is

\[
\ell_{e_{ij}} = \ell_{f_{i}} + \ell_{d_{j}} \tag{33}
\]

The heuristic information for the coupling matrix is

\[
(\eta_{ij}) = (1/e_{ij}, -\ell_{e_{ij}}) \tag{34}
\]

In this case, a real assignment problem with large size is taken into consideration. The problem is the optimum allocation of services in the offices of a multinational company located in Milan, Italy, as described in [14]. The size of the problem is 33.

The problem is then solved using different values in \( \lambda \). The results are shown in Table III and are drawn in Fig. 9.

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>ACO</th>
<th>FACO ( \lambda = 0.3 )</th>
<th>FACO ( \lambda = 0.5 )</th>
<th>FACO ( \lambda = 0.7 )</th>
<th>FACO ( \lambda = 0.8 )</th>
<th>FACO ( \lambda = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value(man-second per week)</td>
<td>438786</td>
<td>433506</td>
<td>432614</td>
<td>430946</td>
<td>440730</td>
<td>442212</td>
</tr>
<tr>
<td>Fuzzy Level</td>
<td>2.009</td>
<td>2.485</td>
<td>2.4817</td>
<td>2.3837</td>
<td>2.301</td>
<td>2.301</td>
</tr>
</tbody>
</table>

The above results are better than those found in [14], where ACO was applied to solve this problem; the value of the objective function is 438786 man-seconds per week. The problem is then solved using different values in \( \lambda \). The results are shown in Table III. It is noted that the proposed algorithm gives better values for TSP and QAP, than those given by the conventional ACO. Furthermore, the range of the
parameter $\lambda$ that gives better values is $\left( \frac{1}{3} \leq \lambda \leq \frac{2}{3} \right)$. Beyond this range, the solution deteriorates.

Taking $\lambda$ greater that $\frac{2}{3}$ means that the assumed probability density function covers a range from $P$ to greater than $P(1 + \ell_r)$. Therefore, any probability density function that covers range from $P$ to greater than $P(1 + \ell_r)$ is not reasonable.

VIII. Conclusions

In this paper, a new FACO technique has been developed. The main advantage of the proposed technique is its ability to represent the uncertainties of the parameters of both the optimization problem and the metaheuristic algorithm in a fuzzy logic-based form. Consequently, the proposed FACO has the ability to give the optimal solution in a form of an optimal value and its corresponding fuzzy level. The fuzzy level of the solution is shown to be interpreted as the uncertainty in the value of the optimal solution. The proposed technique has been tested using two classes of combinatorial problems. The results have been compared to other techniques found in the literature. This comparison indicates that the developed FACO gives better optimal values. Furthermore, the proposed technique achieves the optimal solution in number of trials less than those required by the conventional techniques. This means that the proposed technique has improved the quality of the solution and decreased its time. It is seen that the new concept has an unlimited scope of generalizations and extensions to many classes of problems and systems in various disciplines. A brief list of the areas of further research is presented as follows.

i) Studying of using nonlinear probability density functions on the performance of ACO technique.

ii) Applying the developed technique to other ACO algorithm such Elitist ant system, Rank based ant system and $MAX-MIN$ ant system.

iii) Applying the new fuzzy logic based representation to other combinatorial problems.

iv) Applying the new fuzzy logic based representation to other metaheuristic optimization techniques as particle swarm, tabu search…etc.

REFERENCES


