A Simulation Model of Fluid Flow and Streamlines Induced by Non-Uniform Electric Field

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Abstract- A new model for investigating the non-uniform electric field and potential distribution of fluid flow and streamlines induced by non-uniform electric field with the induced charge in the electrical double layer on the electrode surfaces is presented. Termined AC electro-osmosis, the flow has been simulated numerically using linear analysis. Accurate computation of the non-uniform electric field is a prerequisite for observing the fluid flow and streamlines. The electric field distribution is obtained from Laplace’s and Neumann’s equations. Finite Element Methods is adopted for this work. The simulation results have been compared with available experimental observations of the fluid flow profile obtained by superimposing images of particle movement in a plane normal to the electrode surface. These experimental streamlines gives a good agreement with the simulation model.


I. INTRODUCTION

Electric field plays an important role in the dielectrics and conducting medium; treeing in solids, streamer and electrification in liquids, streamers in gases, and fluid flow in electrolytic solutions [1-2]. The application of a non-uniform electric field to an electrolyte using coplanar electrodes results in steady fluid flow [3]. The flow has its origin in the interaction of the tangential component of non-uniform field with the induced charges in electrical double layer on the electrode surfaces [4].

Electro-osmosis is associated with fluid motion driven because of the presence of fixed or free charges on solid surfaces. In the presence of a DC field, these charges migrate to induce an electro-osmotic flow. However, in an AC field, the mobile charges will simply oscillate in response to the electric field, which results in net fluid motion. Dielectrophoresis, on the other hand, can be performed in an AC field. This is because of the induction of charges that occurs when a body, submerged in a fluid of dissimilar permittivity, is subjected to an external field. The induced charges at the surface rearrange the distribution of mobile ions in the surrounding electrolyte to form an induced electric double layer which will interact with the external electric field and cause a net fluid motion even in the case of an AC field. This induced net fluid motion is termed as AC electro-osmosis [3].

In AC electro-osmosis, the surface charge itself varies with the external electric field. This leads to a similar variation in the charge inside the double layer which when coupled with the electric field produces a net motion of the double layer ions, electro-osmosis, in the AC field. Thus, a more realistic modeling of dielectrophoresis in real media, where free ions are present in the solution, should account for the induced double layer effects. Electro-kinetic forces are produced by the interaction of non-uniform electric fields with polarisable particles [5], providing a method for controlled movement. Since the forces in electro-kinetics are generated by the application of an electric field, the strength and direction of the field are required for the analysis of the available experimental results. Although the bar electrode array has a simple geometry, neither the electric potential nor the field has an analytical expression. An analytical approximation for the potential and forces in the electrode array has been demonstrated using series expansion, both using Green’s functions [6] and Fourier series [7, 8]. It should be noted that both of these are approximations to a geometry for which an analytical representation has not been determined. Numerical methods, such as point charge, charge density, finite difference and integral equation methods have been used to determine electric fields and Dielectrophoretic forces from electrode arrays [9, 10].

This paper presents a simulation model of fluid flow and streamlines induced by non-uniform electric field caused by coplanar electrodes having same voltage with different polarities, Case1, or with same polarities, Case 2, using Finite Element Methods applying the Laplace’s and Neumann’s equations at the electrodes and boundary surfaces. The obtained results have been compared with available experimental observations given by others.

II. EXPERIMENTAL MODEL

In non-uniform electric field generated by coplanar microelectrodes produce steady fluid flow in electrolytic solutions. The fluid moves from the high strength field regions on the edges of the electrodes onto the surface of the electrodes, with the highest velocities found at the edge [3]. The flow has been characterized in terms of a number of experimental variables and is dependent on the frequency and amplitude of the applied signal and on the electrolyte conductivity. The velocity goes to zero at high and low
frequency limits and is maximum at a frequency that depends on the conductivity and position on the electrode surface [3].

This characteristic frequency, \( f_c \), is much smaller than the charge relaxation frequency of the bulk electrolyte

\[
f_c = \frac{\sigma}{2\pi \varepsilon}
\]

where \( \sigma \) and \( \varepsilon \) are the electrical conductivity and permittivity of the electrolyte.

Consider two coplanar electrodes separated by a thin gap, subjected to an AC potential difference and covered in an electrolyte as shown in Fig. 1.

![Schematic Diagram](image)

**Fig. 1.** A schematic diagram of the mechanism of AC electro-osmosis for the experimental electrode array, consisting of two long plate electrode separated by a narrow gap [3].

Figure 1, shows the induced charge layers and the electric field \( E \) at a point in time, resulting from a potential difference applied to the two electrodes. The electric field has a tangential component \( E_x \) at the surface of the electrodes, producing a force \( F_c \) on the charges at the surface. The time averaged value of this force for an alternating potential is nonzero, producing the steady fluid flow pattern shown schematically in fig.1-b . The fluid flow is driven at the surface of the electrode, moving out across the surface and dragging fluid down in the center of the gap. The induced charge accumulates in the diffuse double layer with a sign opposite to the electrode charge. This induced charge is subjected to the action of the tangential component of the electric field, giving rise to a force directed from the center of the gap onto the electrode surface. This force drives the fluid at the level of the electrodes and has a direction that is independent of the sign of the electrode potential. It should be noted that the mechanism requires a non-uniform electric field thus ensuring that a tangential field component exists in the diffuse double layer on the electrodes. The concentration gradients arising from electrode reactions produce a distribution of free charge adjacent to the electrode; this charge interacts with any lateral electric field generating the electrohydrodynamic flow [3].

The solution for the AC electro-osmotic driven fluid flow is calculated numerically using the finite element method. This numerical solution was compared by available experimental results given by [3], the streamlines were recorded as shown in Fig. 2.

![Experimental Setup](image)

**Fig. 2.** A schematic diagram of the experimental setup given by [3].

Figure 2, a schematic diagram of the experimental setup. The electrodes were fabricated up to the edge of the substrate and a glass chamber was constructed around the array, with a vertical glass plate at the end of the electrodes. A microscope objective and camera were then placed horizontally looking along the gap between the electrodes. The camera was then focused at a point inside the chamber at sufficient distance from the electrode ends so that the fluid flow was moving in the vertical plane of focus only. The two coplanar bar electrodes, 0.5 mm wide, 2 mm long and 120 nm thick, with their long edges parallel and separated by 25 \( \mu \)m, the electrodes can be considered to be perfectly polarizable in the range of applied voltage (0 – 2.5 \( V \)), and frequencies ranging from \( 10^2 \) to \( 10^5 \) Hz. This frequency range is always below the charge relaxation frequency \( f_c \) which is from \( 10^6 \) to \( 10^8 \) Hz for the experimental conductivities [3].

The electrolytic solution used were aqueous solutions of potassium chloride with conductivity 2.1 mS/m. Particle tracks indicate that the fluid moves from the center top of the image, down to the center of the gap and out across the electrodes as shown schematically in Fig. 1.
III. NUMERICAL MODEL

As previously stated, for sufficiently low frequencies, i.e., \( f \ll f_e \), the double layer is in quasi-equilibrium [11]. Under these conditions, the bulk electrolyte behaves in a resistive manner and the double layer in a capacitive manner. As a result, the potential in the bulk electrolyte satisfies Laplace’s equation

\[
\nabla^2 \Phi = 0.0
\]

(2)

With the boundary condition just outside the double layer on the electrode surface given by

\[
\sigma \frac{\partial \Phi}{\partial y} = \frac{\partial q_{DL}}{\partial t}
\]

(3)

where \( q_{DL} \) is the charge per unit area in the double layer.

In this equation the lateral currents along the double layer are negligible. Equation (3) describes the charging of the double layer due to the bulk current. The relationship between the charge and the potential drop across the double layer depends on the model used. If the voltage drop across the diffuse double layer is sufficiently small there is a linear relationship between the charge and the voltage, i.e., \( q_{DL} = C_{DL}(\Phi - V) \), where \( C_{DL} \) is the capacitance per unit area of the total double layer and \( V \) is the potential applied to the electrode, and equation (3) can be written with complex amplitudes as

\[
\sigma \frac{\partial \Phi}{\partial y} = i\omega q_{DL} = i\omega C_{DL}(\Phi - V)
\]

(4)

Numerical calculations were performed using partial differential equations (PDE solutions) under MATLAB toolbox. In general, a two-dimensional (2D) problem space is divided into triangular elements and the variables are approximated by second or third order polynomials in each element, see Fig. 4.

Owing to the symmetry of the electrodes, the problem space can be restricted to one electrode of the two with appropriate symmetry boundary conditions. The inner edge of the electrode is 12.5 \( \mu m \) from the symmetry boundary, giving an inter-electrode separation of 25 \( \mu m \). The height of the solution space is 200 \( \mu m \), corresponding to the height of the fluid filled chamber; the length of the solution space was much larger than the gap and the electrode [3].

The electric potential satisfies Laplace’s equation (2), with the following boundary conditions. On the surface of the electrodes, equation (4) satisfies. In addition, there is a boundary condition of odd symmetry \( \Phi = 0 \) on the plane \( x = 0 \) and a Neumann boundary condition \( \partial \Phi / \partial n = 0 \) everywhere else.

IV. NUMERICAL RESULTS

Figure 5, describes the contour lines for real and imaginary parts of the potential, corresponding to a non-dimensional frequency of 8 and applied voltage amplitude 2 \( V \).

For three dimension representation of electrical potential streamlines, Fig. 6 represents the 3D visualization of case one of same voltage with different polarities on the two coplanar electrodes; also Fig. 7 represents the 3D visualization of case two of same voltage with same polarities on the two coplanar electrodes. From Fig. 6 and Fig. 7 the direction of the fluid flow can be recognized and represents the main idea of the fluid particles streamlines shown in the experimental results given by [3].
Fig. 5. The electrical potential calculated from simulation program. The applied voltage to the electrode R1 was 2V and to the electrode R2 was −2 V.

The electric field simulation is given in Fig. 8, taking case one of different voltage polarities on the two coplanar electrodes. From this figure the direction of the fluid flow agree well with the tangential electric field simulation with high value between the two electrodes at the edge facing end.

Fig. 6. 3D dimensions of the electrical potential lines, as calculated from simulation program. The applied voltage to the electrode R1 was 2V and to the electrode R2 was −2 V.

Fig. 7. 3D dimensions of the electrical potential lines, as calculated from simulation program. The applied voltage to the electrode R1 and R2 was 2 V.

Fig. 8. Electric Field Distribution along the gap for case one of different voltage polarities.

Fig. 9. The tangential component of electric field as a function of nodes location along the finite element Discretization.

Figure 9, describes the variation of the tangential component of the electric field simulation as a function of nodes number and nodes location. The nodes arranged as a function of electrode location i.e., the first node was at the electrode edge. From this figure the large value of the tangential component of the electric field which causes the forces of fluid flow direction is adjacent to the electrode surface.
V. COMPARISON WITH AVAILABLE EXPERIMENTAL RESULTS

Figure 10, gives a comparison of the numerically computed streamlines for simulated coplanar electrodes and the available experimental streamlines for the three frequencies used in the experimental observations: (a) 100, (b) 300, and (c) 1000 Hz [3].

The simulation program given in Figs. 5, 6, and 7 show the predicted streamlines, at frequency of 100 Hz and applied voltage 2V. According to this calculation, the drag of the liquid by the surface stresses produces a roll on top of each electrode. The liquid moves faster near the inner edges of the electrodes, where the electric field is stronger and changes more sharply as given in Fig. 9. It can be seen that the numerical computation correctly describes the size and shape of the rolls and the position of the center, the distance of which to the edge decreases as the frequency increases. This agreement is evidence of the AC electro-osmotic origin of the experimental fluid motion given by [3].

CONCLUSIONS

This paper introduces a new approach to simulate the electric field and potential distribution of fluid flow induced by non-uniform electric field. Accurate computation of electric field is a pre-requisite. A simulation model of non-uniform electric field caused from coplanar electrodes was used. Finite Element methods for potential solution and electric field calculation are adopted for this work. The obtained results have been compared with experimental observations. Which give a good agreement with the experimental observation of fluid flow and streamlines given by others.

REFERENCES